Integrated Mathematics + Computer Studies – Gr. 10

Reforming Secondary School Mathematics Education

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IN BRIEF

Wonderful things happen when Grade 10 Mathematics and Grade 10 Computer Studies are offered as an Integrated Program (same students, same teacher, double period, double credit):

- Math is modelled dynamically with computer code ⇨ concepts come to life
- Integration covers curriculum more efficiently ⇨ time for project-based learning
- Collaboration occurs naturally ⇨ sense of community & common purpose

This program was offered in 2017-2018 at two secondary schools in the Wellington Catholic District School Board.

Wellington Catholic District School Board — Faith in Education

THE INTEGRATED EXPERIENCE

The Integrated Program of Grade 10 Mathematics and Grade 10 Computer Studies was offered as a double-period, double-credit, semestered course, taught by the same teacher to the same group of students, with each student having personal access to a laptop for in-class and at-home use.

Teachers noted: "It gives us the flexibility to combine two different topics in a workspace we're passionate in. Giving students the opportunity to express themselves with coding and link it to various concepts in math is a very exciting opportunity."

Computational modelling

The Mathematics + Computer Studies Integrated Program puts a focus on computational modelling of math concepts and relationships. Our society has grown in complexity due to technology and today computational tools are used to model phenomena and processes, to make scientific progress and to succeed economically. Most fields have a computational side: computational biology, computational mathematics, computational finance, computational medicine, to name a few examples.

The authentic *computational modelling* practices of scientists and professionals involve solving real-world problems and building knowledge – to learn – through computational "conversation" and "interaction" with their field (Barba, 2014) "with and across a variety of representational technologies" (Wilkerson-Jerde, Gravel and Macrander, 2015, p. 396). "It's a source of power to *do* something and figure things out, in a dance between the computer and our thoughts" (Barba, 2016).

Concepts come to life

"Integrating computational modelling with mathematics (and with other subjects) prepares students for future success. It also provides a powerful learning tool for designing, testing and refining ideas and understanding concepts" (Gadanidis, Hughes, Namukasa & Scucuglia, forthcoming).

Teachers noted: "It gives students an excellent chance to master a lot of the mathematics concepts taught in the course. Being able to tell a computer how to do those same problems that you've been working out by hand is very powerful. … It also has a great impact on student engagement."

> You think: 'I'm never going to use this.' Then you go into coding and you actually use it. It's more like math in action.
> I like doing math with coding. You get to use math in different ways, to understand it more. You can't code it unless you really understand the math part.
> It feels like there's more space. You don't have to do it like everyone else.
> It lets you go in depth. You see how each function affects the next. How it connects together. *Student comments*

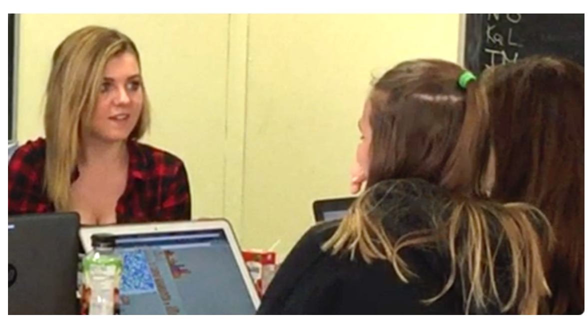
Students preferred the integrated approach: "In regular math class, the teacher teaches you everything. But with coding you're more independent. It's more of a sense of accomplishment. … I used to like math. I understood it. But it got boring, doing the same thing over and over. At one point you're thinking, 'I don't want to do this anymore.' … You would get an equation and you would use it, but you didn't really know what it meant: you just used it because that's what you were told to do. … Everyone is just in their own desk, looking at their own work. … Like in a jail cell."

Sense of community

Students noticed a sense of community in the learning environment: "I didn't take this course expecting it to be more collaborative. It just happened. Naturally. I like it. … It's more of a group feeling in the atmosphere, asking questions and trying to understand. … It feels good to know if you're stuck you can turn to someone else for help. … There are different ways to solve a problem, and sometimes you come up with a totally different way than others. … It's great that more girls are taking this course. I think most of the girls in our class are probably better than most of the boys. … It has helped me with my collaboration. I'm more open to work with people."

> Working in small groups, it's interesting to see students of a variety of abilities in computer studies and math able to help one another and work collaboratively to solve problems. It allows every student to demonstrate their aptitude.
> It also allows students to be more independent. Working on a program, errors are immediately fed back to you, and you are able to recognize and correct them, as opposed to waiting for a teacher. *Teacher comments*

Visitors noted: "The classrooms appear to be very dynamic and students seem more than engaged. When looking at it from the outside, one might say that it looks like a chaotic experience, because there's so much conversation, students are turning back and forth to each other, they are getting up on their feet freely, and it seems like a more unstructured learning environment than what we regularly see when you walk down the halls of high schools."



Teachers noted: "I think such an unstructured environment promotes creativity and flexibility. If you look at software development environments, there's a lot more room for an informal atmosphere, for different ways of thinking, different solutions to the same problem. … We want to turn the teacher role more into a facilitator role. … It's a safe environment where students feel a lot more comfortable to ask each other questions, and not feel that they might ask the wrong types of questions."

Project-based work

Teachers noted that the Integrated Program helps cover curriculum expectations more efficiently and create time for project-based work: “We are able to deliver the math curriculum while embedding coding concepts. It makes it much more efficient in terms of how fast we can cover curriculum and give them more time to understand it. … Students working with both math and computer science at the same time allows us to hit certain goals a lot faster. … This creates time for passion projects and independent projects. … With two periods back-to-back, we don’t have as many interruptions.”

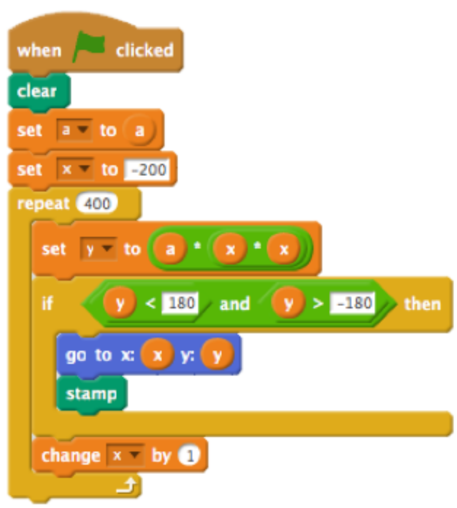
> I enjoy the projects. I liked building a game and getting it to work. It was pretty interesting to see everyone’s ideas.
> I like the satisfaction of finishing or doing a project. I did that. It’s my work. It’s a great feeling.
> It would be cool to create something that’s going to help someone else.
> I learned that it is possible to learn pretty much anything. *Student comments*

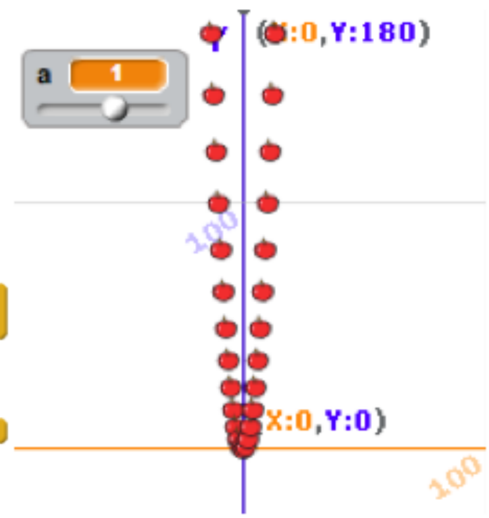
CURRICULUM EXAMPLE: MODELLING FUNCTIONS

Using a variety of coding environments, students dynamically modelled functions.

Using Scratch (http://scratch.mit.edu)

Scratch uses visual coding blocks that even young children can easily use. The code plots the function $y = ax^2$. How could you edit the code to plot $y = a(x – h)^2 + k$?





Using Python (http://cemc.uwaterloo.ca/console)

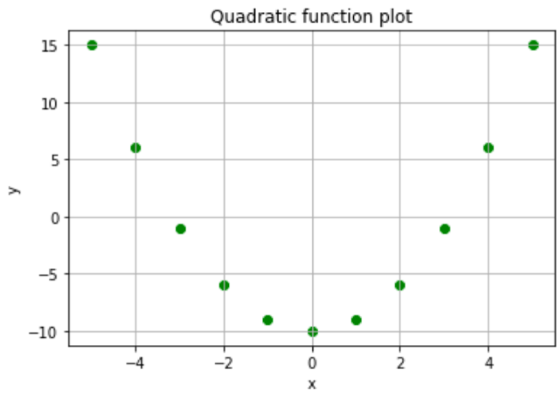
Python from the University of Waterloo uses text-based code and output. The code lists (x, y) pairs for the function $y = x^2$. How could you edit the code to also print differences between successive y-values? What pattern emerges? What is the sum of the first 5 odd numbers? The first 10?

```
# (x, y) values of a function
for x in range (0,11):
    y = x**2
    print (x,y)
```

```
0 0
1 1
2 4
3 9
4 16
5 25
6 36
7 49
8 64
9 81
10 100
```

Using Jupyter Notebook (http://jupyter.org)

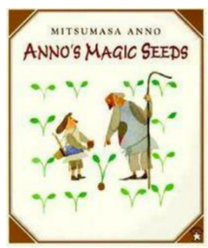
Jupyter Notebook is a powerful web application used by computational scientists, which allows you to create and share documents that contain live code, equations, visualizations and narrative text. This code models the quadratic function $y = x^2 + k$. What is the role of k in the graph of the function?

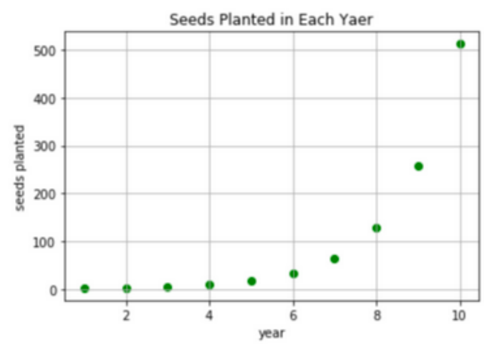



```
# Quadratic functions

# initialize
k = -10 # value of k
xArray = [] # x-values
yArray = [] # y-values

# define function
for x in range (-5,6):
    y = x*x + k # quadratic function
    xArray.append(x) # append x-values to xArray
    yArray.append(y) # append y-values to yArray

# print x array
print("x-values:", xArray)

# print y array
print("y-values:", yArray)

# scatter plot
plt.scatter(xArray,yArray, color = 'g')

# plot labels
plt.xlabel('x')
plt.ylabel('y')
plt.title('Quadratic function plot')
plt.grid(True)

plt.show()
```

Modelling Civilization (http://mathsurprise.com/code/civilization)

Extending their work with linear and quadratic functions, and using a mathematical story (Anno, 1995) as context, students investigated how computational mathematics may dynamically model "civilization."





NEXT STEPS

The Grade 10 Mathematics + Computer Studies Integrated Program will continue in 2018-2019 at the Wellington Catholic District School Board. Pedagogical ideas and resources developed for computational modelling of mathematics concepts and relationships will be shared in professional development sessions for secondary school mathematics teachers.

MORE INFORMATION

Coding for All: a story of purpose (Wellington Catholic News)
http://www.wellingtoncdsb.ca/BoardOffice/Policies/Documents/WCDSB 17-15 WCN8_final_web.pdf

KNAER Mathematics Knowledge Network (Computational Thinking in Mathematics Education CoP)
http://mathnetwork.ca/ct

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