Proceedings of the 2020

Online Seminar Series on Programming in Mathematics Education

(June 19, July 3, 17 & 31, August 14 & 28)

Buteau, C., Gadanidis, G., Gannon, S., & Figov, A. (Editors)
(January 2021)

Available at http://mkn-rcm.ca/online-seminar-series-on-programming-in-mathematics-education/

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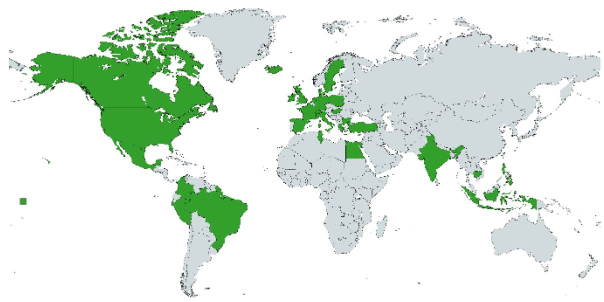
About the Seminar Series

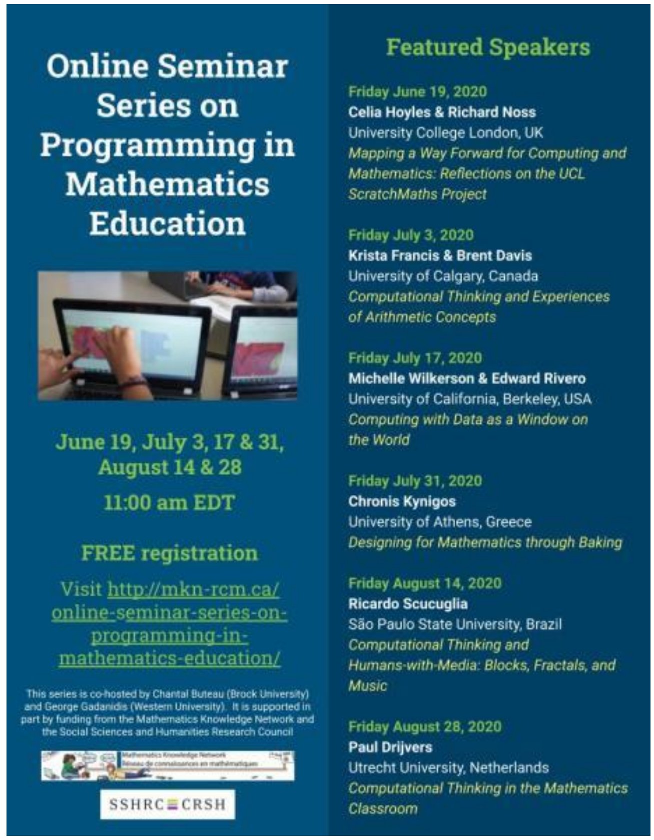
The *Online Seminar Series on Programming in Mathematics Education* was launched as a response to the cancellation or postponement of various scholarly events, such as the *International Congress in Mathematics Education (ICME) 2020* in Shanghai, China, as a result of the COVID-19 pandemic situation. We decided to fight the confinement by instead meeting and sharing ideas in an online setting.

Our series, held online during the Summer of 2020, aimed to provide a platform to discuss recent research on the integration and use of programming (or computational thinking more broadly) in mathematics programs. Research presented during the series addressed topics including the interplay between the affordances of computational thinking and mathematics, the nature of exemplary tasks, pedagogical models, instructional materials and resources, and assessment practices.



The seminar series included 261 registered attendees from both a Canadian and global audience, including 6 Canadian provinces (British Columbia, Alberta, Saskatchewan, Ontario, Quebec, and New Brunswick) and 30 countries across 5 continents (North and South America, Europe, Africa, and Asia). Participants included faculty/researchers and post-doctoral fellows, 95 graduate and undergraduate university students, 53 elementary and secondary school teachers, 4 government or regulatory agency employees, and 13 others.

Bi-weekly seminars featured presentations by nine researchers from six different countries. Beginning the seminar series, Celia Hoyles and Richard Noss (University College London, United Kingdom) presented the UCL ScratchMaths project, a two-year mathematics and coding curriculum for 9 to 11-year-old students aligned with England’s mandatory computing and mathematics programs. In particular, they highlighted the overlap between mathematical and computational thinking as found in ScratchMaths’ curriculum. Krista Francis and Brent Davis (University of Calgary, Canada) presented a research study about students’ conceptual development of 'numbers' through robotic coding tasks. They discussed a theoretical basis in conceptual metaphor and conceptual blending theories for programming in mathematics education, which grounded their detailed analysis of a teaching episode video involving a teacher and two Grade 4 students. Next, Michelle Wilkerson and Edward Rivero's (University of California, Berkeley, USA) seminar emphasized the connection between computational thinking and data literacy, and the transformative role of data and stories. The researchers discussed an activity implemented in a middle-school class in which students engaged in selecting and manipulating real data concerning nutrition, and critically explored disagreements between the data and their own experiences.



In his seminar, Chronis Kynigos (National and Kapodistrian University of Athens, Greece) discussed the "half-baked" microworlds and artifacts of MaLT2 and ChoiCo. He focused his presentation on various constructionist activities in which programming and mathematics are interrelated in multiple ways. Ricardo Scucuglia's (São Paulo State University, Brazil) presentation then highlighted three research episodes focusing on humans-with-media and aesthetic mathematical expression. His discussion emphasized different affordances and aspects of computational thinking involving the arts in mathematical activities at the elementary, secondary, and university levels. Finally, Paul Drijvers (Utrecht University, Netherlands) highlighted mathematics tooling and the connections between mathematics education theory and computational thinking education. He discussed some of these connections through the design of secondary mathematics activities. Drijvers also included an overview of all presentations as the series' concluding speaker, integrated in this summary.

Participants were able to engage in rich discussions with the speakers and conference organizers following each seminar presentation. Feedback from participants included the following comments:

- "I believe it was a very strong seminar! Congratulations to the organizers and to the speakers!" (Brazil)
- "Thank you very much for planning, preparing, and creating this opportunity to hear interesting lectures." (Iceland)

- "I think the topics of the seminars are very interesting. It provides me with much new knowledge. So glad to join this seminar" (Indonesia)
- "Well done. Glad to see people from different perspectives and different countries." (Canada)
- "I wish to thank all the organizers for making these seminars available worldwide." (France)
- "Very informative and appreciate the cycle of learning that reflects our local context. Looking forward to delving into their resources and sharing with colleagues." (Canada)
- "Thank you for organizing this seminar and the whole online series. The presentations have been clear and well-driven for all participants." (Spain)
- "Congratulations for a fabulous series of online seminars." (Mexico)

Recordings of all six seminar presentations are available on the series website (http://mkn-rcm.ca/online-seminar-series-on-programming-in-mathematics-education/), along with associated online resources, instructional materials for teachers, and related research papers. As of January 2021, there have been a total of 547 views of the seminar recordings.

The *Online Seminar Series on Programming in Mathematics Education* was funded in part by the Mathematics Knowledge Network (MKN, http://mkn-rcm.ca/), hosted by the Fields Institute for Research in Mathematical Sciences (http://www.fields.utoronto.ca/) and funded by the Ontario Ministry of Education, and the Social Sciences and Humanities Research Council of Canada (https://www.sshrc-crsh.gc.ca/).

We wish to thank all seminar speakers and series participants for their enriching contributions to the seminar series.

From left to right: *Online Seminar Series on Programming in Mathematics Education* co-hosts Dr. Chantal Buteau (Brock University) and Dr. George Gadanidis (Western University), and series coordinators Sarah Gannon (Brock University) and Arielle Figov (MKN).

Mapping a Way Forward for Computing and Mathematics: Reflections on the UCL ScratchMaths Project

Professor Dame Celia Hoyles & Professor Richard Noss
UCL Institute of Education
University College London
United Kingdom

In our talk, titled **Mapping a way forward for computing and mathematics: Reflections on the UCL ScratchMaths project** (available at http://mkn-rcm.ca/online-seminar-series-on-programming-in-mathematics-education/), we presented the context of the research. We began by showing a glimpse of the way computing was introduced in schools in England culminating in 2014 with the introduction of a new statutory primary National Computing Curriculum for students aged 6 to 16 years. This curriculum included as a key aspect that students should *design*, *build* and *debug* programs.

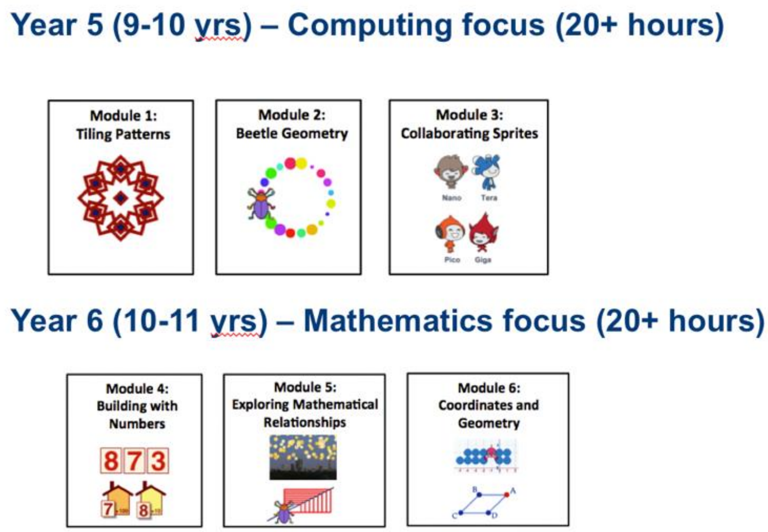
We then moved on to outline the history of programming and mathematics, which for us had its roots in innovations from MIT in 1980s, the vision of Seymour Papert for the development of Logo, the setting up of the Logo Mathematics group and the subsequent 50 or more years of research into implementation across the world (see for example Papert, 1972; Hoyles & Noss, 1992; Noss & Hoyles, 1996; and Monaghan, Trouche, & Borwein, 2016).

We described our ScratchMaths (SM) project, which designed and implemented a longitudinal two-year intervention at the intersection of mathematics and computing, targeted for 8–11-year old students in English schools and involving programming in Scratch. Our team was interdisciplinary with expertise in mathematics education, computing, and design, and we worked closely with teachers to iteratively develop our original designs. We aimed to foster *mathematical thinking*; that is, an awareness and appreciation of mathematical structure, the articulation of coherent explanations for outcomes and the reasoning behind them, and being comfortable and fluent with the formal expression of relationships.

To pursue this aim we developed student and teacher curriculum support materials organized into six modules, three to be taught per year, involving about 20 hours teaching. The modules can be considered as *microworlds*, designed to provoke engagement with key ideas in mathematics and in computing (for background on microworld development, see Hoyles, 1993; and more recently Kynigos, 2020, *Online Seminar Series for Programming in Mathematics Education* lecture available at http://mkn-rcm.ca/online-seminar-series-on-programming-in-mathematics-education/).

Figure 1 shows an overview of the six UCL ScratchMaths microworlds:

Figure 1: *Overview of UCL ScratchMaths Microworlds*



All the materials are freely available, now updated to Scratch 3.0, through the UCL website (http://www.ucl.ac.uk/scratchmaths)

We presented what we took as the components of computational thinking, based on the large number of rather similar definitions and resources available at that time (for background, see Benton et al., 2017, and for an up-to-date summary of definitions and research on Computational Thinking, see Drijvers, 2020, *Online Seminar Series for Programming in Mathematics Education* lecture available at http://mkn-rcm.ca/online-seminar-series-on-programming-in-mathematics-education/).

We summarise the definition we adopted and sought to operationalise as:

- **Abstraction:** seeing a problem and its solution at many levels of detail
- **Algorithms:** thinking about tasks as a series of logical steps
- **Decomposition:** understanding that solving a large problem can involve breaking it down into smaller problems
- **Pattern recognition:** appreciating that a new problem is likely to be related to other problems already solved
- **Generalization:** realizing that a solution to a problem can be made in ways that can solve a range of related problems

We discussed the design principles underpinning ScratchMaths and in particular its pedagogic framework iteratively designed with teachers in four "design" schools. This consisted of the "5 E's," as follows:

- *Explore*: Investigate, try things out yourself, debug in reaction to feedback
- *Envisage*: Have a goal in mind, predict outcome of program *before trying*
- *Explain*: Explain what you have done, articulate reasons behind your approach to yourself and others

- *Exchange*: Collaborate and share, try to see a problem from another's perspective as well as defend your own approach and compare with others.
- *bridgE*: Make explicit links to the mathematics curriculum

These principles framed the professional development (2 days per year) that was part of the SM intervention and the planned classroom implementation. We presented some exemplar activities to illustrate our approach. For an earlier summary of our project, see Noss, Hoyles, et al., 2020, in press.

The results of the ScratchMaths intervention are reported in full in the evaluation report to be found at: https://educationendowmentfoundation.org.uk/projects-and-evaluation/projects/scratch-maths/. This report was the result of an *independent* evaluation specified by our funders. We note that ScratchMaths had a positive and significant impact on student computational thinking (CT), as reported by the evaluator using a randomized control trial methodology with 111 schools across England and measured by a test of computational thinking designed and administered by them at the end of the first year of the intervention. We also note the important results that this positive effect was particularly evident among educationally disadvantaged students. There was no evidence of any interaction between the impact of SM on CT test scores and gender: thus, girls and boys appeared to engage with SM to a similar extent, an outcome that is particularly important in view of the finding persistent in the literature that girls tend not to be as engaged in computing as boys.

However, there was no impact of SM on mathematics attainment as measured by the independent evaluators on the basis of the student results in the statutory national mathematics test (Key Stage 2 test) taken by all 11-year-old students in England. As a way to seek to explain these findings, in the talk we called on the notion of *fidelity* of implementation (see O'Donnell, 2008, for a review of Defining, Conceptualizing, and Measuring Fidelity of Implementation), and how in our study fidelity appeared to have been negatively influenced by the high-stakes testing in mathematics in England, leaving little room for innovation in classrooms for 11-year-olds. These tests involve a formal paper-and-pencil mathematics test and are used to rank schools and teachers, so much time is spent reviewing and revising, thus teachers found little resource to devote to ScratchMaths.

Finally, we reflected on the implementation of ScratchMaths and how it could be improved in future work, not least as teachers are becoming more confident and competent in their understanding of computational concepts, in teaching them, and in using them to explore mathematical ideas through programming. We pointed to the use of ScratchMaths; for example in Australia (see Holmes, Prieto-Rodriguez, et al., 2018), and in a nationwide project in Spain (Final report of "Escuela de Pensamiento Computacional", School for computational thinking, 2020, part of which concerned a replication of our work along with assessing the impact of ScratchMaths. We translated one finding from this report that was of particular relevance to our talk: namely, it was reported that "the results show that it is possible to include programming activities in 5th grade in the area of mathematics, so that students not only learn to program and engage in computational thinking, but also improve the development of their mathematical competence greater than their colleagues who have worked in this same area using other types of activities and resources not related to programming."

Reflections at the End of our Seminar and After

At the end of the seminar, we posed some research challenges that might be interesting for others in the community to address. These challenges included the need to:

1. Develop more *nuanced, rigorous, and targeted assessment instruments of student (and maybe teacher) content knowledge in mathematics, mathematical thinking and in computing* to be administered as post-tests following engagement in each microworld and as delayed post-tests several months later, rather than use the standard national tests as adopted in the evaluation of UCL ScratchMaths.
2. Research in more detail the *actual* practices in classrooms to include documentation of teacher and student interactions and output, in order to provide detail of classroom implementation and how far the pedagogic framework was enacted. In particular such research might provide some explanation of the outcomes reported for UCLScratchMaths in relation to socially disadvantaged students and girls as mentioned above, taking as a starting point the idea of fidelity while recognizing the 'chaotic' nature of real classrooms, teacher practices, and policy demands.
3. Develop a more detailed description of the *nature and content of the professional development* that is undoubtedly needed prior to successful implementation of the ScratchMaths intervention.

At the time UCL ScratchMaths was conceived and operationalised, computing was still new in England. Teaching and learning has been transformed in the intervening years, not least as a result of the coronavirus pandemic. Teachers and students have undoubtedly become more fluent in working online in general and in programming in particular. One might expect that the integrity of the SM materials would remain constant while its implementation would be less challenging. But this is a matter of further research.

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Additional Resources

Links to all resources can be found on the series website (http://mkn-rcm.ca/online-seminar-series-on-programming-in-mathematics-education/).

1. Module 5 – Investigation task for exploring mathematical relationships (Year 6) for teachers
2. Video journal of an Ontario graduate student (now teacher) using the Module 5 task
3. Research papers related to the seminar:
    - Making Constructionism Work at Scale: The Story of ScratchMaths
    - Computational Thinking for Mathematical Understanding: A Case Study of Exploiting Programming to Explore Face Value
    - Beyond Jam Sandwiches and Cups of Tea: An Exploration of Primary Pupils' Algorithm-Evaluation Strategies
    - Bridging Primary Programming and Mathematics: Some Findings of Design Research in England

Computational Thinking and Experiences of Arithmetic Concepts

Krista Francis & Brent Davis
University of Calgary
Canada

Abstract: In this paper, we discuss how students experienced number while learning to program their robot to move. First, we will provide overview of the context and the research by describing a task used to develop conceptions of "number." Then, we will introduce two discourses from the cognitive sciences that orient the work: Conceptual Metaphor Theory (Lakoff & Nuñez, 2000) and Conceptual Blending Theory (Fauconnier & Turner, 2003). Finally, we will analyze an interaction among two students and their teacher as they tacitly negotiate meanings of number that are appropriate to the task of programming their robot to move forward 100 cm. Our analysis suggests that computational settings may afford rich settings for experiencing and blending distinct instantiations of a range of number concepts in manners that support flexible and transferable understandings.

Keywords: Programming robot, number, Grade 4, elementary mathematics

Learning Discourses

Learning Discourses in Education (Davis & Francis, 2020) analyzes, critiques and sorts/organizes over 850 discourses on learning according to their core foci and implicit metaphors. One of the strategies used to highlight confluences and disjunctions among discourses is a map, the horizontal axis of which distinguishes between correspondence discourses (which assume radical separations of internal from external, self from other, individual from collective, etc.) and coherence discourses (which reframe dichotomies as heuristic conveniences, while embracing evolutionary dynamics within and across nested systems). The map's vertical axis is used to locate discourses according to their relative emphases on the nature of learning ("interpreting learning," the lower region) and advice for teaching ("influencing learning," the upper region). We situate all our work among coherence discourses, and the research reported lands in the lower region of the map (on making sense of learning) with, we believe, strong implications for the upper region (on informing teaching).

A secondary organizational strategy used on the map is the clustering of similarly themed discourses. The work described here fits most strongly with the cluster that has been labeled "Association-Making Strategies," which collects a variety of currently popular research foci – such as spatial reasoning, conceptual metaphor, varied modes of reasoning, and ranges of cognitive bias.

Celia Hoyes and Richard Noss (2020) situated their work within constructionism, like many other researchers with interests in computational thinking (e.g. Abelson, 1981; Buteau, 2019; Papert, 1993). We see constructionism as profoundly complementary to our interests. On the Learning Discourses map, we have located it directly above the "Association-Making Strategies" cluster, meaning that we interpret it to share similar theoretical commitments and metaphoric frames, but with a stronger focus on implications for teaching.

With the Association-Making Strategies cluster, we find two discourses – Conceptual Metaphor Theory and Conceptual Blending Theory – to be especially useful for our current efforts to combine research interests in computational thinking and learning arithmetic. Reflecting a core insight from recent decades of cognitive sciences research, Conceptual Metaphor Theory (Lakoff & Johnson, 1999) is based on the principle that human thought is mainly analogical and associative rather than logical and deductive. Consequently, conceptual metaphor theory looks at metaphor as a core tool of human thinking. The theory examines how metaphor makes it possible to understand one conceptual domain – that is, idea, cluster of related experiences, set of interrelated interpretations – in terms in terms of another conceptual domain. It also examines how metaphoric associations among domains can orient perception, prompt action, and serve as uncritical justifications for further interpretations. Metaphor is core to human thinking and is especially important for bridging bodily experience to abstract constructs.

Conceptual Blending Theory (Fauconnier & Turner, 2003a, 2003b) extends conceptual metaphor theory in the suggestion that complex concepts and creative leaps typically involve blends of multiple metaphors. Such processes are seen as core aspects in human thought and language – pervasive, constant, and largely nonconscious. Once blends have been made, they become resilient and invisible for the knower. A ready example is the concept of "number" – which, for most adults, operates as a seamless blend of count, size, rank, distance, location, and value. Typically, adults find it difficult to see these interpretations of number as different. By contrast, young learners may initially experience them as distinct and incompatible.

Grounding Metaphors of Arithmetic

Lakoff and Núñez (2000) identified four grounding metaphors of number: *object collection*, *object construction*, *measurement*, and *object along a path.* The metaphor of *arithmetic as object collection* is based on a one-to-one correspondence of numbers to physical objects. With this metaphor, numbers are understood as counts, and they differ from one another in terms of *how many*. The metaphor of *arithmetic as object construction* frames number in terms of size that differ from one another in terms of *how large*. The *measuring stick* metaphor maps numbers onto distances, and so numbers differ from one another in terms of *how long*. The metaphor of arithmetic as *object along a path* is based on location, through which numbers are different by virtue of their locations. While the importance of these metaphors for mathematical understanding may not be immediately obvious, Lakoff and Núñez (2000) argued that the development of robust understandings of each and the capacity to move nimbly among them is critical for the emergence of mathematical understanding. Table 1 below translates Lakoff and Núñez's (2000) grounding metaphors of arithmetic into metaphors of number.

Drawing on Conceptual Blending Theory (Fauconnier & Turner, 2003a), Davis (2020) worked with teachers to identify three additional metaphors for number that arise in blends of Lakoff & Núñez's four grounding metaphors. The resulting total of seven core metaphors of number for elementary school mathematics is presented in Table 1. Between *number as count* and *number as size* are two blends: *Number as rank* blends notions of count and location, making it possible to answer questions of "Which?" by making available the ordinal numbers, *Number as amount* blends notions of count and size in order to render large numbers accessible. The third blend, *number as reification*, collects all the other instantiations. This consolidated instantiation is able to operate without a referent. That is, for example, five is simply 5 – not 5 things, the 5th thing, 5 large groups, size 5, an interval of 5, or location 5. Simply – but complexly – 5.

Table 1. *Four metaphors of number associated with Lakoff and Núñez's (2000) four grounding metaphors of arithmetic*

Lakoff and Núñez's Grounding metaphor	Associated metaphor of number	Matter addressed (situation modeled)	An instantiation of '5'
OBJECT COLLECTION	NUMBER AS COUNT	How many?	
OBJECT CONSTRUCTION	NUMBER AS SIZE	How big?	
USING A MEASURING STICK	NUMBER AS LENGTH	How long?	
MOTION ALONG A PATH	NUMBER AS LOCATION	Where?	

Designs for Complementing Mathematics Learning

An integrated, well-blended conception of number is not automatic. For young learners, it is likely that grounding metaphors of number are initially engaged individually, and the learner may at first experience some difficulty reconciling different metaphors. In contrast, an expert is likely to move seamlessly among metaphors, and experienced knowers many even find it difficult to parse well-consolidated conceptions. That is, the experienced knower might not be able to make distinctions among elements that the young learner must struggle to connect. To make such connections, learners must encounter different instantiations of number at the same time.

For the past decade, we have been designing robotics tasks that, we believe, do just that. While our original motivations in moving to robotics settings were to participate in the growing interest in computational thinking and to examine their potential contribution to the development of spatial reasoning, one of the immediate, consistent, and striking realizations in working with robotics was the manner in which even simple tasks supported learners' understandings of number, number systems, and computation, along with the development of computational fluency.

Early on, it became clear to us that the number line is a critical element in supporting learners' consolidations of the concept of number. In Lakoff & Núñez's (2000) terms, we see the number line as a sort of "linking metaphor" – a construct that enables extensive bridging across domains of experience, potentially yielding sophisticated, abstract ideas. Lakoff & Núñez (2000) suggested that linking metaphors require explicit teaching; for the most part, they are inventions, not part of one's natural world, that are designed for specific conceptual purposes. The task described below, along with the analyses of the engagements around the task, were developed with this in mind. We sought to design a task that deployed the number line as a site to bring together multiple interpretations of number in manners that compelled learners to integrate those interpretations by grappling with varied entailments.

Robotics (Coding Motion) Focus: A Hypothesertion

We speculate that coding/computational-thinking environments – **and working with robotic motion in particular** – are superb spaces to develop senses of number and number sense. This is largely because multiple instantiations of number are typically invoked, usually simultaneously – and might be anticipated, given that computational thinking is an offspring of mathematics.

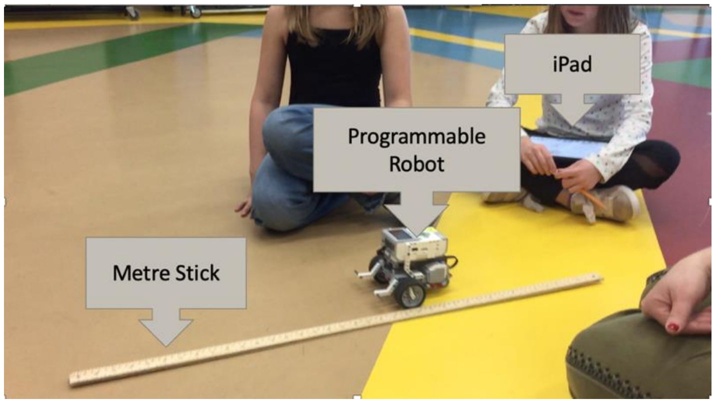
Research Setting

Context

The study took place in a local non-profit, independent K–12 school in Calgary, Alberta that specializes in working with students with learning differences. Weekly robotics classes were offered during regularly scheduled mathematics classes for all students in Grades 4, 5, and 6.

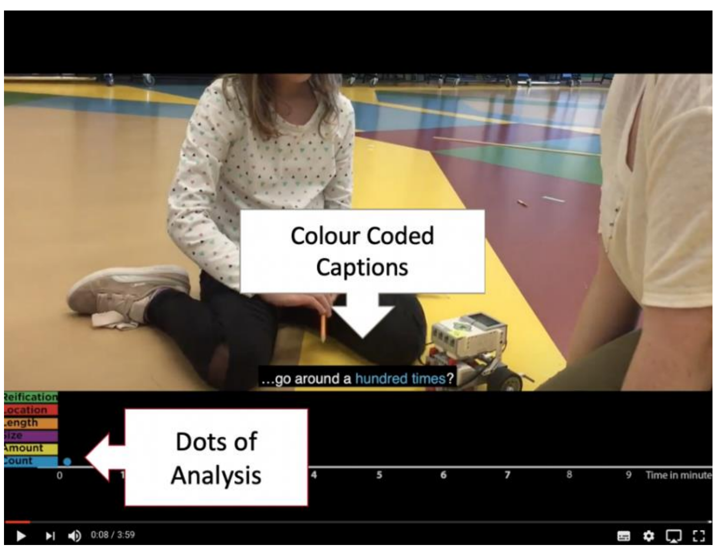
The video data highlighted in this paper were taken in a Grade 4 classroom at the beginning of the year. The students had not yet formally encountered decimal numbers, and they were just starting to program the movement of their robots. The question that oriented the activity captured in the episode was, “How many wheel rotations are needed for the robot to travel 100 cm?” With regard to equipment, each pair of students as a metre stick and used their iPad with EV3 Mindstorms software to program their EV3 Mindstorms robot.

Figure 1. *Two Grade 4 Students and Their Teacher Programming Their Robot to Move 100 cm.*



This 10-minute episode was selected through an exhaustive interpretive selection process (Knoblauch, 2013) of more than 240 hours of video recordings, based on the quality and focus of the action. Transcriptions of the video were imported into NVivo and we coded the videos together based on the instantiation of Table 1. The videos were then edited to include color coded captions and in-time analysis of the video. As you view the videos, watch for these color-coded captions and dots of analyses (see Figure 2).

Figure 2. *Color-Coded Captions and Dots of Analysis in the videos*



Minutes 00:00–03:59 – How far does the robot travel? https://vimeo.com/313928391

The recording begins with the students thinking that 100wheel rotations will be needed for the robot to travel 100 cm. The teacher shows them how far the robot travels with one-wheel rotation by aligning the pointed wheel cap vertically down and moving the robot forward by pushing it along with her hand, and she asks them to guess again. The students try 15-wheel rotations. Their next try is 7, and it is much closer to the desired distance. The students appear to be quickly gaining a sense of what a *count* of 1-wheel rotation means as a *length*. In this 4-minute portion of the episode, five different metaphors of number are invoked: in order of frequency, they are: *count (14), size (4), length (6), location (7), and reification (1).* With regard to the tracking of metaphors at the bottom of the screen, it can be observed that *count*, although dominant at the opening, was quickly abandoned – suggesting that the participants realized on some level that it was relevant, but not especially useful for solving the task.

Minutes 04:00–05:32 – How far does the robot travel with 7? https://vimeo.com/317354442

The second clip starts with the trio observing where the robot stops (*location*) with seven wheel rotations. The girls *count* each rotation as the robot travels alongside the ruler. The teacher asks, "Is seven too much …" (*amount*) "… or too little?" (*size*). Gabby responds with, "too much" (*amount*). They then find that six stops even closer, but still moves past the desired end point (*location*). The girls reason that they "needed to go back 1" (*location*) to five. In this video, the use of the instantiation of location becomes amplified. While references to *length* disappear, our suspicion is that those are conflated with *location*, en route to a more consolidated notion. In the 1.5 minutes of this clip, there are 14 number references across five different metaphors: *location* (7), *count* (1), *amount* (3), size (1), and *reification* (2).

Minutes 05:33–08:40 – Are there numbers between 5 and 6? https://vimeo.com/319520044

Recall that this episode represents the students' first formal encounter with decimal numbers. Notice their body language, starting at about 28 seconds into the clip, when the teacher asks them if there are numbers "between 5 and 6"? The students nod their heads "no," and to our observation, their faces suggest questioning and lack of understanding. At this point, the teacher cycles through multiple instantiations, as though searching for something that resonates. She starts by using money (*amount*) to talk about decimal and common fractions. She then moves to *location*. In the three minutes of the clip, there are 38 utterances of number across four different metaphors: *count* (3), *amount* (9), *location* (11), and *reification* (15). Notable in this clip are, firstly, the pronounced shift to *reification* – which signals to us both a further consolidation of multiple instantiations and an enhanced interpersonal accord on that emerging consolidation – and, secondly, a shift from discrete to continuous notions of number. On these matters, it seems appropriate that the teacher invokes the blend *amount*, which sits across discrete and continuous metaphors.

Minutes 08:41–09:43 – Video 4: Homing in on an appropriate metaphor
https://vimeo.com/325933850

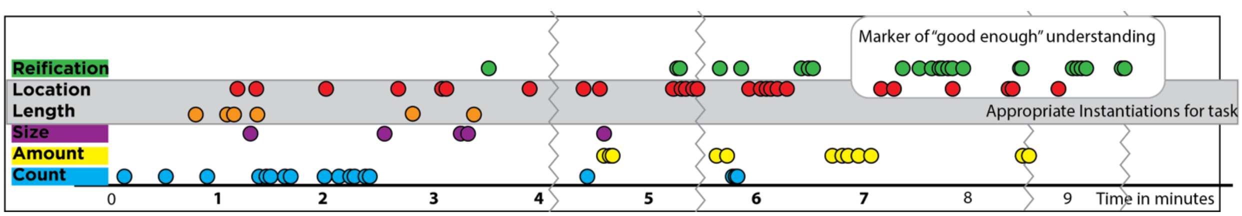
In final clip, the pair of students use 5.7 as their final try. The teacher finishes the discussion with an instantiation of *location*. In the minute-long clip, there are seven utterances of number across two different metaphors: *location* (1), and *reification* (6).

Notably, in the week after this episode, this same pair of students used decimal numbers in another context – fluently, appropriately, and without prompting.

Summary

Across this episode, there were 91 instantiations of number (see Figure 3). Textbooks have very few instantiations. As illustrated in Figure 3, the metaphors that proved to be appropriate for the task are *location* and *length*, and the strong presence of *reification* was an indicator of adequately shared understandings. We suspect this sort of pattern – that is, the shift to a single metaphor, coupled to significant usage of *reification* – is a common and important marker of good-enough common understanding … or, in more fraught encounters, total bafflement.

Figure 3. *Use of Metaphors in Conversation about Robot-Measurement Task*



Final Thoughts

We have used this video episode in several situations. Of note, when viewers are not alerted to attend to metaphors of number, few observers notice the somewhat incoherent barrage of interpretations that fly around in the first five minutes. By contrast, when asked to attend to metaphor (even when not provided with the analysis presented in Figure 3), viewers tend to notice that barrage without much difficulty.

With regard to task design, our suspicion is that the number line is a critical feature for blending multiple instantiations. With the number line, one can simultaneously *count* spaces, compare *sizes*, determine *lengths*, and identify *locations*. As we attempt to illustrate with Figure 4, this simultaneity of instantiations offers more than elaborated spaces for interpretation. They also afford access to new types of number and number systems. Rational numbers, for example, are readily discussed in terms of only *__counts__*, but are readily accessible with blends of *counts* and *sizes* – and even more readily accessed when a well-parsed number line affords blends of *counts*, *sizes*, *lengths*, and *locations*.

It thus goes without saying that coding robot motion is likely to be a powerful learning space, with regard to number concepts. In our experience, it has been particularly powerful for rational numbers, and especially decimal fractions, for students in upper elementary. We routinely encounter learners whose understandings of number are clearly fragmented and whose abilities to manipulate decimal fractions are highly procedural. Yet, consistently, even preliminary encounters with programming robots to move have proven to be powerful sense-making spaces, as learners emerge with demonstrably greater fluency with varied applications of number. To re-emphasize, our strong suspicion is that the built-in number lines of such encounters are core to their effectiveness – and, thus, an important focus for task designers interested in supporting arithmetic learning while promoting computational thinking.

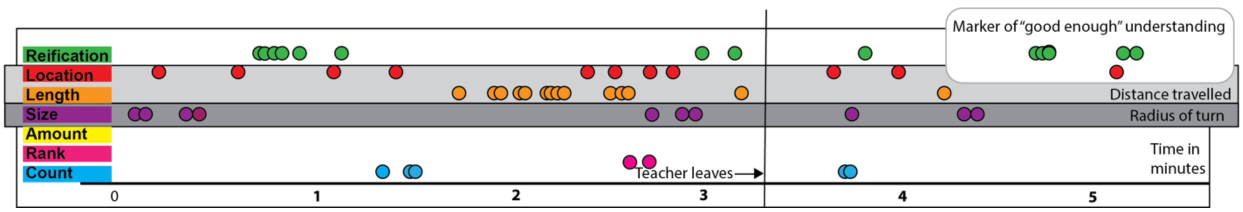
Figure 4. Seven metaphors of number, along with some illustrative entailments (from Davis, 2020)

Metaphor, Metonym, or Metaform of Number		Matter addressed (situation modeled)	Associated Grounding Metaphor(s) of Arithmetic	An instantiation of '5'	How 'less' and 'greater' tend to be expressed: less	greater	How addition tends to be seen	Some encounters/ contexts/ uses	Numbers made available
COUNT	quantity	*How many?* (discrete)	OBJECT COLLECTION		fewer	more	combining sets	counting; sorting; clustering	Whole N°s; Natural N°s; cardinals
RANK	quantity	*Which?* (discrete)	OBJECT COLLECTION		ahead	behind	changing rank	sequencing; ranking; grading	ordinals
AMOUNT	quantity	*How much?* (discrete, but experienced as continuous)	OBJECT COLLECTION & CONSTRUCTION		less	more	pooling amounts; amassing	pricing; accounting; apportioning	large numbers; discrete fractions
SIZE	magnitude	*How big?* (continuous object)	OBJECT CONSTRUCTION		smaller	larger	growing; joining pieces	assembling; sharing; ratios	continuous fractions
LENGTH	magnitude	*How long?* (continuous dimension)	USING A MEASURING STICK		shorter	longer	extending; moving farther	scale-based measuring; traveling	Rational N°s; Irrational N°s; Integers
LOCATION	entity	*Where?* (discrete site in continuous space)	MOTION ALONG A PATH		left of (or, lower)	right of (or, higher)	shift in location	locating; scheduling; reading time	Real N°s; Imaginary N°s; Complex N°s
REIFICATION	entity	*What?* (disentangled from physical instantiations)	All of the above	5	<	>	binary operation	symbolic manipulation; computing	Any/all of the above

To repeat an earlier point, none of this should be surprising. Coding is an offspring of mathematics; it always already involves powerful and sophisticated conceptual blends of concepts. When coding is combined with motion, orienting attention towards the number line can provide insights into number. The number line is perhaps the most powerful instantiation for number, and coding motion supports rapid familiarization, robust understanding, and flexible usage.

Effective pedagogy in enabled by nuanced pre-understanding of which instantiations to invoke when. In another recent study, a teacher was briefly informed of the instantiations of number invoked with coding motion (link here). His awareness of knowing when to invoke which instantiation prompted him to be more deliberate in his conversations, contributing to clearer communication about and quicker resolution to a more complex task. While the full analysis is not yet complete, we are able to offer a summary chart (see Figure 5). We leave it here as a provocation, and we invite the reader both to use it to follow the linked video and to contrast it with the trace presented in Figure 3. We believe it serves as further confirmation of our hypotheasertion on the potential contributions of coding motion to learning arithmetic.

Figure 5. *Use of Metaphors in Conversation about Robot-Steering Task*



Author Note

This research was supported in part by funding from the Social Sciences and Humanities Research Council.

Krista Francis orcid.org/0000-0002-8268-405X

Correspondence concerning this article should be addressed to Dr. Krista Francis, 2500 University Dr. NW, Calgary, AB. T2N 1N4. kfrancis@ucalgary.ca

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Additional Resources

Links to all resources can be found on the series website (http://mkn-rcm.ca/online-seminar-series-on-programming-in-mathematics-education/).

1. Seminar presentation slides
2. Rich mathematical robotic tasks (Grades 4 – 9) for teachers
3. Research paper elaborating further the ideas presented in the seminar

Sociocritical Literacies and Computing with Data as a Window on the World

Michelle Wilkerson & Edward Rivero
University of California, Berkeley
United States of America

In their foundational and prescient 1996 book "Windows on Mathematical Meanings," Noss and Hoyles explored how computational and Constructionist approaches to mathematical thinking and learning could not only foster the mathematical development of students, but also reshape what mathematical cultures look like. Mathematics, they noted, formed the basis of computing—governing the smart phones and laptops that have become all too ubiquitous in our world. As such, these same devices could also offer a window—or indeed, many windows—into mathematics and the ways that students made sense of it. Such windows, they argue, could grant power as "[t]he realm of social and cultural life, as much as the quality of life of the individual, is impoverished by the absence of mathematical windows to understand and interpret the world, and to change it" (p. 2).

Computing with data (e.g. computational statistics, data visualization, data science) has become one such window that we, as mathematics educators and as a society more broadly, trust will provide students with leverage to understand, interpret, and change the world. We see this in calls for data literacy education, and in the increasing prominence of data not only in computing and mathematics education, but indeed across the curriculum (Engel, 2017; Finzer, 2013). It is important, however, to recognize that computing with data, like any social endeavor, is not a purely good or even neutral endeavor (Philip, Schuyler-Brown, & Way, 2013). The relationships between students, data, computing, and data cultures, and the extent to which data literacy can actually be experienced as empowering by students, is further mediated by broader social and power relations (Van Wart, Lanouette, & Parikh, 2020). Here, we describe our early work toward theory and curriculum to explicitly support the development of what we call *sociocritical data literacy*, which takes these broader relations into account through computational data activities in middle school classrooms.

Sociocritical Data Literacies

On first thought, one might imagine a window on the world suggests that an observer, perhaps a student, is standing on one side of the window observing the world on the other. But we are all *part* of the world, and inextricably a part of the world's systems—environmental, economic, and ideological. When we look through a window, we see only a *part* of the world, dictated by our position within it, as well as a reflection of ourselves. And windows are not perfectly transparent. They can and do distort, and it is becoming increasingly apparent that computing with data has created harmful distortions of the world. These distortions emerge in various forms including surveillance, anti-Black racism, and corporate attention hoarding; we encounter them daily in internet search results (Noble, 2018), and people's futures are at stake as they manifest in health and crime statistics and prediction algorithms (O'Neill, 2016).

These challenges behoove mathematics and computing educators to focus not only on data literacy and the computational skills it entails, but also on sociocritical forms of data literacy. A growing number of scholars are calling for this type of broader education that focuses not only on data computing itself, but also the cultural and ideological contexts in which data computing is practiced, and for what ends (Philip, Schuyler-Brown, & Way, 2013; Vakil, 2018). Our own conceptualization of sociocritical data literacy builds on Gutiérrez's (2008) notion of sociocritical literacies, whereby students reorganize components of both school-based and everyday literacies into means to enact social transformation and critical social thought. In this conceptualization, sociocritical literacies privilege and are contingent upon students' sociohistorical lives, as they intersect with academic practices and topics. A distinguishing feature of a sociocritical literacy is its attention to contradictions in and between texts, institutions (e.g., the classroom, the academy), and sociocultural practices, locally experienced and historically influenced.

Applying this notion of sociocritical literacy to computing with data involves juxtaposing students' personal, cultural, and community-based understandings of the world with data-based representations. In doing so, students may observe and examine the reasons for inconsistencies between their own experiences and statistical trends; identify contradictions in whether and how students and their communities might be represented (or not) through data collection and analysis; and seek to understand whose interests are served when a particular set of data are collected or analyzed. These juxtapositions reveal how students' lives intersect with society, and how these intersections may be reified or challenged through the collection, analysis, and sharing of data. Returning to the window metaphor, these juxtapositions are intended to highlight the ways in which data, as a window on the world, might be distorting, highlighting, or obscuring students, their communities, and their experiences of the world—as well as revealing possible ways to address those distortions or omissions.

"Data Moves" as a Way to Engage with Sociocritical Aspects of Data

We have been working to develop sociocritical data literacies with middle school students through the development of computer-based, multi-unit science curricula through a project called "Writing Data Stories". This project is a collaboration between UC Berkeley (with Kris Gutierrez), the Concord Consortium (with Bill Finzer), and North Carolina State University (with Hollylynne Lee), funded by the U. S. National Science Foundation (IIS-1900606). It includes the development of new pedagogical frameworks, curricula, and software to position young people as "architects of data" (Stornaiuolo, 2019) that can not only analyze, but transform data to highlight new perspectives within datasets, or structure those datasets toward new investigative goals. Importantly, students tell “data stories” that highlight the nature and purpose for each data transformation they execute, making clear the ways in which goals and perspective shape what data are shared, and inform what inferences can be drawn from data analysis.

The theory that drives our work at the intersection of computing, mathematics, and society focuses on “data moves” (Erickson, Wilkerson, Finzer, & Reischman, 2019). Data moves are actions that alter a dataset’s contents, structure, or values, while maintaining the integrity of the dataset. Normally, this type of data preparation is ignored in the process of analysis (With some exceptions, notably Wild and Pfannkuch, who called these processes “transnumeration”; 1999). However, we argue that it is in this process of reviewing and rethinking the details of a dataset’s structure where issues of justice and ideology emerge. One clear and very recent example of this is in the U. S. data for COVID-19 infections. At first, many states did not report data about race. Researchers aware of the racial disparities in the US knew that risk of exposure and risk of complication were higher among communities of color and pushed for these disparities to shape how data are analyzed. Using a data move we call grouping, this racial perspective provides a new lens on the data that reveals Americans who are Black or African American are 2.5 times more likely to die of complications related to COVID-19 (see the COVID Racial Data Tracker; https://covidtracking.com/race).

We illustrate our approach through an early unit we developed focused on food and nutrition. As we describe above, a key aspect of our approach is for students to understand their personal connections and sociocultural histories as they relate to the topic under study. The food and nutrition unit begins with an exploration of students’ own home food practices, reflection on why their family prefers the foods they do, and an investigation of the ways in which media and advertisements normalize some food choices (e.g. cereals for breakfast) over others. Students also explore how data (e.g. health information, popularity of products, peer ratings, etc.), along with other strategies, is used in argumentation to convince others to make certain food decisions. By making these everyday types of data objects of, we were able to gain a better understanding of how data affects how young people see the world and act on it. Over the course of a few weeks, we saw how young people learned about the ideological nature of data and also brought in everyday knowledge that revealed how they were aware of how data governs their lives. We also gave students opportunities to engage with the strategies companies use to convince and to reauthor data sets to make them consequential to their own lives.

After these investigations, students were then provided with a nutrition dataset that features nutritional data about 70+ breakfast cereals. Rather than take this dataset for granted and analyze it, we asked them what was *missing* that would be important in their own food decision making. Students noted a number of inconsistencies between their own values and practices when it came to breakfast, and what was reflected in the dataset. Why are hot foods underrepresented? Why isn't taste or price taken into account? Why are some cereals that are advertised as healthy high in sugar, while other foods they eat daily not even volunteered as an option? Students added some of their own foods and attributes of interest to the dataset for comparison and decision making, before conducting analysis. In Figure 1 below, one student group has introduced both price and taste as important dimensions in making their food decisions (and there is a column called "cooking", too, which they had not yet filled out). On the left, you can see another set of students has used taste to group their data. They then used multiple graphs (here fiber is featured "behind" the taste graph) to determine the relative health of foods they identified as tasting good or bad.

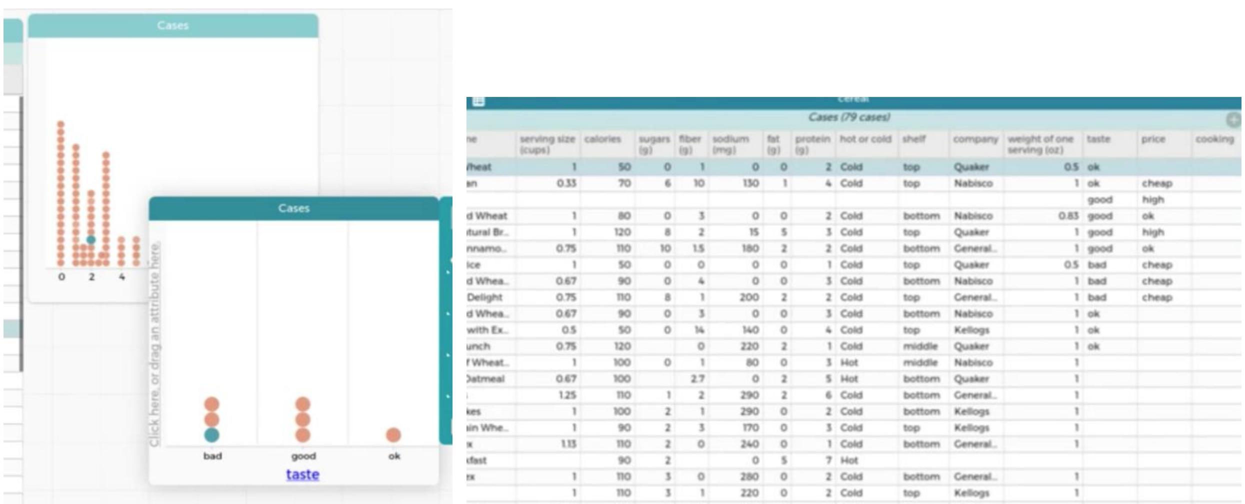

Figure 1. Left, students use graphing tools and their modified nutrition dataset to find foods that are both low in sugar (left graph) and that taste good (right graph). Right, students have added taste, price, and whether a food needs to be cooked as important factors to consider when making food decisions.

Our work with students was cut short by Shelter-in-Place orders related to COVID, but we found early results encouraging and are looking forward to learning more. We have been developing units on climate change and its disproportionate impacts globally, and on environmental racism and the local effects of pollution on respiratory health, which will be publicly available soon.

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Additional Resources

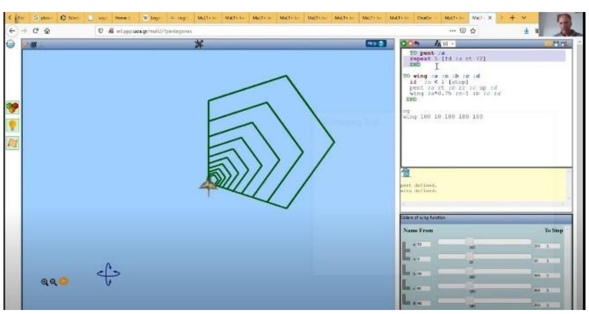
Links to all resources can be found on the series website (http://mkn-rcm.ca/online-seminar-series-on-programming-in-mathematics-education/).

1. Writing Data Stories Public Curriculum, including Introduction, Nutrition, and Climate Units (lessons in English and Spanish)
2. Cereal dataset utilized in the seminar (in English and Spanish)
3. Newsletter article discussing the Writing Data Stories project
4. Research paper elaborating further the ideas presented in the seminar

Designing for Mathematics Through Baking

Chronis Kynigos
National and Kapodistrian University of Athens
Greece

Abstract: 40 years on from Mindstorms, programming is mostly poorly connected to mathematics curricula considered either as irrelevant or at best as a fragmented add-on to an already extended syllabus. In my talk I will focus on my recent attempts to design contexts where constructionist activity fuses programming with the use of mathematical concepts to engage in computational thinking. I will discuss two diverse situations: one where programming animated 3D graphical models addresses the mathematical properties inherent in their behavior (MaLT2) and another where mathematics becomes a humble tool in a very different setting and paradigm, that of addressing socio-scientific issues within a post-normal science paradigm (ChoiCo). Respectively MaLT2 and ChoiCo are authoring systems freely available to use on the web and in English. 'Baking' means modding and fixing specially crafted 'half-baked', i.e. faulty or incomplete, artefacts or games. I will draw examples from three contexts in Greece: a new Masters course at NKUA with students of diverse specialty, an also new Masters course for Swedish Mathematics Teachers through my collaboration with Linnaeus University, and thirdly the Ministry of Education's digital infrastructure including more than 200 programmable micro-experiments in the year 5-11 mathematics curricula.



Additional Resources

Links to all resources can be found on the series website (http://mkn-rcm.ca/online-seminar-series-on-programming-in-mathematics-education/).

1. Seminar presentation slides
2. Online constructionist authoring systems:
   - MaLT2 for mathematical modelling (English manual)
   - ChoiCo for socio-scientific gaming (English manual)
3. Educational Technology Lab at the National and Kapodistrian University of Athens
4. Research papers relating to the seminar:

- Programming Approaches to Computational Thinking: Integrating Turtle Geometry, Dynamic Manipulation and 3D Space
- Modifying games with ChoiCo: Integrated affordances and engineered bugs for computational thinking
- Constructionism: Theory of Learning or Theory of Design?

Computational Thinking and Humans-with-Media

Ricardo Scucuglia
São Paulo State University
Brazil

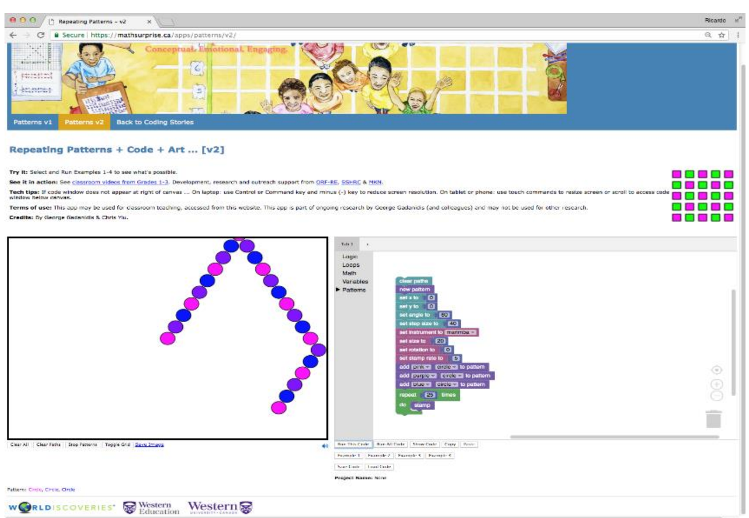
Introduction

We have investigated the following research question: How do students-with-media think mathematically-artistically-computationally?

The agency of technology. Borba and Villarreal (2005) propose that knowledge is not produced only by humans, but by thinking collectives of humans-with-media. This view reveals the protagonist role or the agency of (digital) technology in shaping mathematical activity. We have argued this notion is also related to Papert's (1993) idea about "objects-to-think-with". More recently, we have explored aspects of computational thinking (CT) using humans-with-media as a framework.

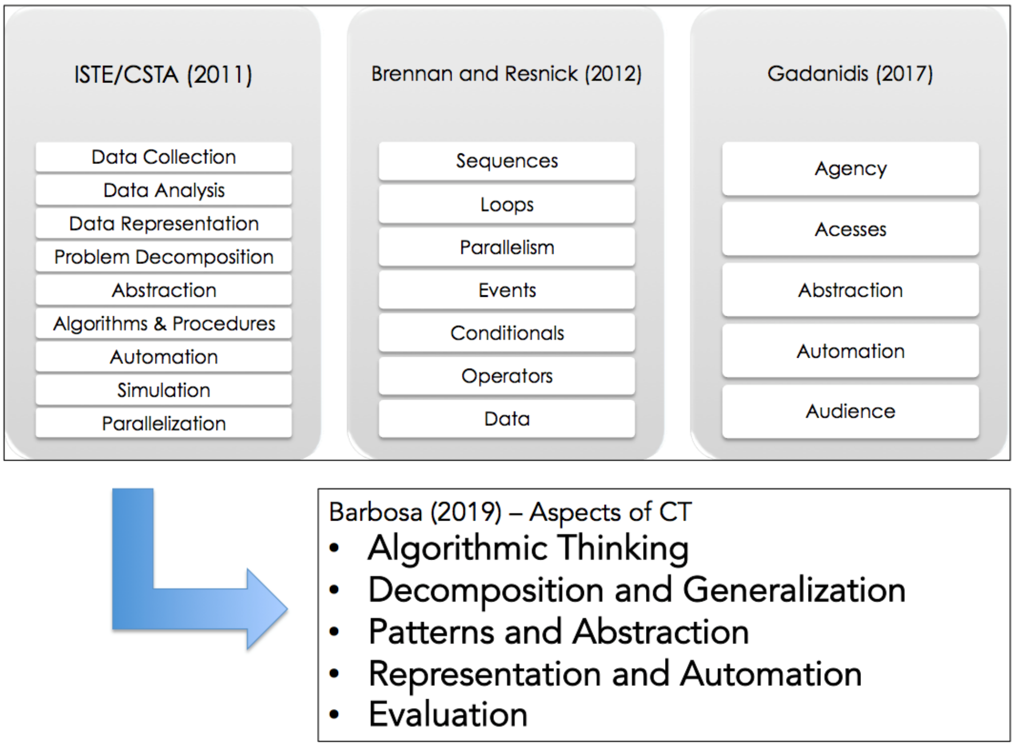
Blocks

We have developed teaching experiments based on the use of an application designed by Gadanidis and Yiu (2017). This application is named Repeating Patterns (see https://mathsurprise.ca/apps/patterns/v2/). In a case involving pre-service teachers in Brazil, we have highlighted: (1) the interface between CT and the arts (patterns and symmetries involving shapes, colors, and sounds); (b) the complexity of thinking in terms of complex connections between representations, and (c) the possibility of multiple solutions in terms of problem solving in CT environments (Scucuglia, Gadanidis, Hughes & Namukasa, 2020).



Aspects of CT and Fractals with GeoGebra

Barbosa (2019) proposes five categories to explore *aspects* of CT. These categories are: *algorithmic thinking*, *decomposition and generalization, patterns and abstraction*, *representation and automation*, and *evaluation*. These categories actually comprise three views on CT: (1) *skills* of CT (ISTE/CSTA, 2011); (2) *concepts* of CT (Brennan & Resnick, 2012); and (3) *affordances* of CT (Gadanidis, 2017).



Barbosa and Scucuglia (2019) used these five categories on *aspects* of CT to investigate how pairs of mathematics majors construct fractals using GeoGebra in teaching experiments. The findings are: (a) the design of the tasks explored 2D/3D fractals (e.g. Sierpinski triangle and Sierpinski tetrahedron) and this offered ways to elaborate different strategies to construct geometric patterns; (b) there is a synergy between the conceptual nature of fractals (self-similarity) and main aspects of computational thinking (e.g. repetition and automation); (c) the use of sliders and the of homothetic tool of GeoGebra offered ways to thinking collectives to gradually explore deeper aspects of CT.

Music Production and CT in mathematics education

We have explored how pre-service teachers create math songs. Music production offers ways to think mathematically, since mathematics is related to music theory (Gadanidis & Scucuglia, 2020). The use of the software Logic Pro X has offered possibilities for mathematics and education majors to think mathematically-musically-computationally, regarding the exploration of several types of representations and patterns (visual, numerical, aural) (Scucuglia, 2020).

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Additional Resources

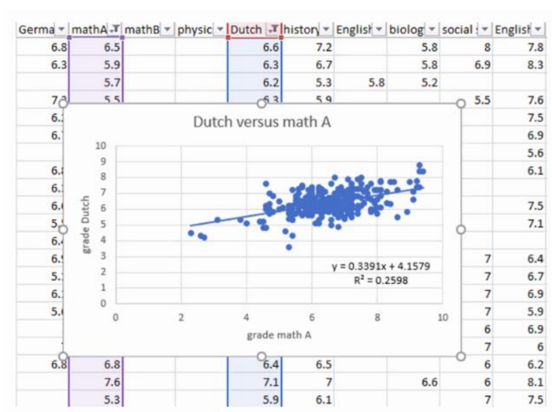
Links to all resources can be found on the series website (http://mkn-rcm.ca/online-seminar-series-on-programming-in-mathematics-education/).

1. Seminar presentation slides
2. GPIMEM (Research Group in Informatics, other Medias and Mathematics Education)
3. GeoGebra-based animations of fractals
4. Related research paper elaborating on the concept of music production and mathematics teacher education

Computational Thinking in the Mathematics Classroom

Paul Drijvers
Utrecht University
Netherlands

Abstract: Nowadays, much attention is paid to the development of students' competences in the field of digital literacy and computational thinking. However, it is not always clear what computational thinking exactly is. Also, as mathematics educators, we may feel some resemblance between higher-order learning goals in mathematics teaching and computational thinking, but still are unsure about how to reconcile the two, and how to address computational thinking in the mathematics classroom. What is computational thinking and how can it be related to mathematics education goals and practices?
To address these questions, I will first reflect on the notions of computational thinking and mathematical thinking. Next, the results of a Delphi study on computational thinking in mathematics teaching will be presented. Finally, I will address the preliminary results of the teaching experiments we carried out in applied and pure mathematics courses for 16-17-year-old students in the Netherlands.



2. What are the common aspects of computational and mathematical thinking?

Agreement upon:
- **decomposition**
- **pattern recognition**
- **abstraction**
- **generalization**
- **algorithmic thinking**
- **logical thinking**
- **analytical thinking**
- **structured problem solving**
- **problem analysis**
- **conceptual understanding**
- **modeling**
- **evaluating strategies**
- **making a plan**