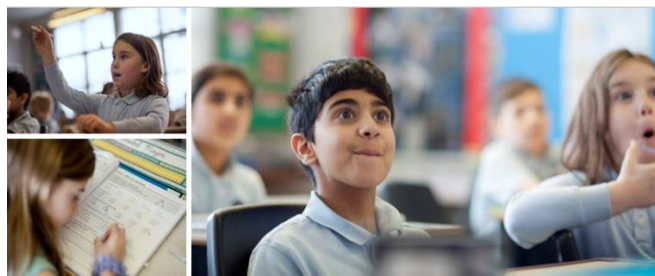


From research to practice: Supporting learners and teachers of mathematics

Summary: In this facilitation guide we provide ideas shared during the Professional Learning Network webinar presented by Dr. John Mighton on February 15, 2022 (organized through Zoom). The guide introduces evidence-based principles of instruction and the results of research and implementation of JUMP Math. Topics suitable for all grades include:

- How to teach Novice vs. Expert Learners? Using “structured inquiry” to engage all students.
- How to support teachers who experience anxiety and lack confidence for teaching mathematics?

The guide contains some Grade 3-6 mathematics problems and the links to digital supports and other resources for teachers, students, parents, and tutors (e.g., online Financial Literacy lessons; free essential lessons for K-8 to mitigate learning loss; core classroom program for K-8; Math Minds—free online course from the Werklund School of Education at the University of Calgary, and more).



Introduction — Features of JUMP Math

JUMP Math is a program of delivery of research-informed classroom materials and professional development resources for teaching mathematics. The program was developed by an award-winning charitable organization dedicated to reducing learning gaps and inequities in mathematics education. The research conducted in Canada, USA, and elsewhere, agrees that this program successfully replaces mathematics anxiety with an understanding and a love of mathematics in both students and educators. The features of JUMP Math, with their research to practice orientation, include confidence building; structured inquiry; scaffolded instruction; continuous assessment; differentiation; and mental math. These features are well aligned with the new Grades 1–8 Mathematics curriculum (2020).

Confidence building¹. As Darling-Hammond et al. (2020) note, when faced with a task, students often wonder if they are capable enough, what supports for them exist in solving the task, and do they feel safe attempting the task. The research in cognition shows the brain can be altered by education if the person is attentive. However, a student’s brain cannot be truly attentive unless the student is confident and excited and believes that there is a point in being engaged in the work. This is especially important to truly support learning of populations of students who are known to have experienced underachievement in mathematics and science (Dweck, 2008). Dweck confirms that, “interventions that change mindsets can boost achievement and reduce achievement discrepancies” (p. 2).

Because attitude and mindset² are important, JUMP Math strives to reduce math anxiety by building on success in small steps. When students who are struggling become convinced that they cannot keep up with the rest of the class, their brains begin to work less efficiently, as they are never attentive enough to fully consolidate new skills or develop new neural pathways. That is why it is so important to give students the skills they need to take part in lessons and to give them opportunities to show off by answering questions in front of their classmates.

Research in psychology and mathematics education highlight the process of concreteness fading (Fyfe et al., 2015; 2014). In both informal and formal learning of mathematics, this process involves learning that the “quantity ‘two’ could first be represented by two physical apples, next by a picture of two dots representing those apples, and finally by the Arabic numeral 2” (Fyfe et al., 2014, p. 11). Therefore, concreteness fading involves using multiple models of mathematical concepts, presented in a gradual specific progression.³

Question: What strategies do you use to motivate students, to help them feel safe, and to be actively involved in learning?

Structured inquiry⁴. For inquiry to be successful, it must be done in carefully scaffolded steps gradually increasing student agency over learning (MacKenzie, 2016). Structured inquiry is a balanced instructional approach that incorporates explicit instruction combined with guided and independent

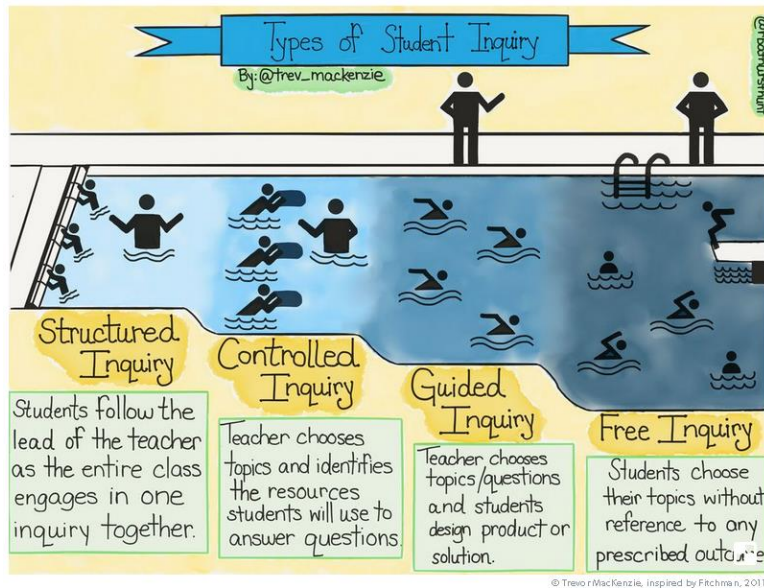
¹ According to the Ontario Curriculum, Grades 1–8: Mathematics (2020), to develop the social-emotional learning (SEL) skills, students need to “develop self-awareness and self-confidence” (p. 36).

² An important set of the 21st century competencies and transferable skills require that “Students learn to think about their own thinking and learning (metacognition) and to believe in their ability to learn and grow (growth mindset). They develop their ability to set goals, stay motivated, and work independently” (p. 41)

³ The curriculum recommends, “implementing research-informed instructional approaches (e.g., Concrete – Semi-Concrete – Representational – Abstract) when introducing new concepts to promote conceptual understanding, procedural accuracy, and fluency” (p. 95).

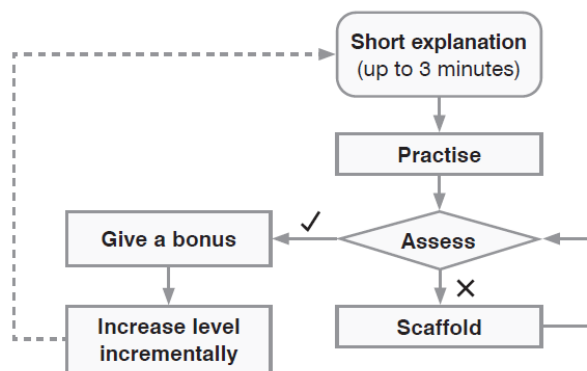
⁴ One of the categories of knowledge and skills across the school curricula is “Thinking and Investigation – The use of critical and creative thinking skills and inquiry and problem-solving skills and/or processes” (p. 57).

practice. In JUMP Math lessons, students are expected to derive concepts and solve problems themselves, but the teacher provides enough rigorous guidance to make sure this happens with all students and not just the advanced few. Students are led to deep conceptual understanding and computational fluency.



Question: How do you implement inquiry in your classes? Are there steps in MacKenzie’s 4-step inquiry model that you are willing to try?

In structured inquiry, teachers offer students purposeful practice that immediately follows the teaching of a skill or scaffolded step of a concept. Teacher covers concepts explicitly only for brief periods before asking a question or assigning a challenge that students can explore independently in pairs or in groups. Throughout this process, the teacher continuously assesses and confirms each student’s understanding and mastery.



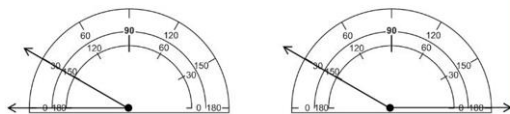
Question: Which aspect of the JUMP Math model of structured inquiry are you willing to try? Why, or why not?

Scaffolded instruction⁵. To develop into a successful problem-solver, one does not have to solve a huge number of problems, but these problems need to be carefully designed and selected (Sweller, 2008). Two factors predominantly interfere during problem-solving: learner’s selective attention and limited cognitive processing capacity. Extraneous information can get in the way of problem solving (Kaminsky & Sloutsky, 2014). Especially younger children easily get distracted by extraneous perceptual information that the learning context offers, “consequently hindering either the learning of relevant structure or its transfer to new situations” (p. 351). Therefore, each JUMP Math lesson offers a series of carefully considered exercises and explorations in which each new concept follows from the last.

Learning challenges need to be scaffolded into manageable chunks. By so doing, educators prevent situations in which the student’s cognitive processes required by the different activities overlap, negatively affecting problem-solving process (Sweller, 2008). Scaffolded instruction is an effective instructional practice where new ideas and skills are reviewed and introduced in smaller incremental steps directly linked to and built upon earlier learning.

Introduce protractors. Give each student a protractor and SAY: This is a *protractor*. Have students examine their protractors and compare them to rulers. Remind students that when they measure with a ruler, they have to line up one end of the object with the zero mark. Have students find the zero mark on their protractors (There are two of them, one at each end!) Point out that a protractor has two scales, both with the same unit, but in opposite directions. A ruler can have two scales, too, but the two scales would use different units. Explain that having two identical scales going in different directions allows you to measure the angles from both sides, but this also means that you need to decide which scale you will use each time.

Project **BLM Simple Protractors** onto the board. Explain that the protractors on the BLM are simplified pictures of a protractor, without all the tiny markings in between the larger angles. Draw the angles below on the board:



Pointing at the first picture, ASK: Is this angle an acute angle or an obtuse angle? (acute) Circle the numbers 30 and 150 that the arm of the angle passes through. ASK: Which one is the

Is this an **acute angle** or an **obtuse angle**?

What **angle** does it show?

Hint: Read from the scale that has a “0” on the arm of the angle.

Question: What strategies do you use to maximize students’ attention and cognitive processing capacity?

Continuous assessment⁶. According to Darling-Hammond et al. (2020), “assessments that place value on growth rather than on scores earned at one discrete moment have been found to create higher motivation, greater agency, and higher levels of cognitive engagement, as well as stronger achievement gains... [thus especially] reducing opportunity and achievement gaps for traditionally marginalized students” (p. 121). The progression of learning contained in each JUMP Math lesson enables active and ongoing assessment, often referred to as “continuous assessment.” Through observation, questioning,

⁵ According to the 2020 curriculum document, “Teachers select instructional strategies to effectively introduce concepts, and consider how they will scaffold instruction in ways that will best meet the needs of their students” (p. 9).

⁶ “The primary purpose of assessment and evaluation is to improve student learning” (p. 45). Led by this idea, teachers provide “ongoing descriptive feedback that is clear, specific, meaningful, and timely to support improved learning and achievement” (p. 46).

and interaction with students as they work, teachers are readily able to check for understanding, identify and correct misunderstandings, and differentiate instruction with timely interventions. Continuous assessment often provides a more accurate and complete picture of a student's ability and has a positive impact on learning.

Question: What aspects of the mastery-oriented performance-based assessments you find effective in your practice?

Differentiation⁷. Each child is unique, having “individual needs and trajectories that require differentiated instruction and supports to enable optimal growth in competence, confidence, and motivation” (Darling-Hammond et al., 2020, p. 98). Also, novice learners are different from expert learners. JUMP Math recognizes that classrooms are diverse in their makeup and needs, and that students range in their levels of focus, concentration, engagement, processing speed, and readiness to explore and demonstrate learning. To support this diversity, JUMP Math lessons and supporting resources provide multiple approaches to exploring, practising, and assessing skills. Teachers can differentiate the development of skills without differentiating the outcomes of their students by teaching lessons in short instructional pieces and assigning scaffolded exercises, hands-on activities, and extension questions. In JUMP Math, mastery is the goal of each lesson, and all students are supported to reach that goal. In addition to paying attention to each child, the years of implementation have convinced us that children can experience a sense of awe or excitement collectively.



Question: What is your vision of teaching that is inclusive of all students?

⁷ In the 2020 curriculum, “Differentiation is planned as part of the overall learning design, but it also includes making adaptations during the teaching and learning process based on ‘assessment for learning’. Common classroom strategies that support differentiated instruction include cooperative learning, project-based approaches, problem-based approaches, and explicit instruction. Unless students have an Individual Education Plan with modified expectations, what they learn continues to be guided by the curriculum expectations and is the same for all students” (p. 9).

Mental math⁸. Price, Mazzocco, and Ansari (2013) report that fluency in basic mental arithmetic plays the fundamental role in the acquisition of higher-level mathematical competence, even at the tertiary school level. Mental math is a mathematical framework that includes number sense, computational fluency, and the application of number concepts through purposeful and varied practice, not just rote memorization. In JUMP Math, basic knowledge is valued, and mental math fluency is regularly practiced.

$$2 \times 6 = 12$$

$$4 \times 6 = 24$$

$$6 \times 6 = 36$$

$$8 \times 6 = 48$$

Question: Do you find mental math skills useful even for older students?

To conclude, research in cognitive science has shown that students learn much more efficiently when teachers:

- ✓ provide adequate review,
- ✓ avoid cognitive overload,
- ✓ help students develop mental representations by directing their attention to the salient features of the problem,
- ✓ scaffold challenges into manageable chunks, varying only one or two elements of a problem at a time,
- ✓ provide adequate practice to consolidate concepts, and
- ✓ raise the bar incrementally to generate engagement.

Research about JUMP Math

Math Minds ⁹ is a research partnership between JUMP Math, the Werklund School of Education at the University of Calgary, and several school districts in Alberta. In a five-year study, students roughly doubled their percentile rankings on the conceptual understanding and problem-solving scales of the *Canadian Test of Basic Skills*.

In the second year of a randomized controlled trial conducted by the University of Toronto and the Hospital for Sick Children (Solomon et al. 2019), grade 3 students made significantly more progress in problem solving than students in the control group (with a large effective size of .54 in the high-fidelity

⁸ Mental math is featured in the 2020 mathematics curriculum as a strategy to be practiced in school, but also in different everyday out-of-school situations, such as during grocery shopping, playing math-based games and puzzles; using different “opportunities for mental math estimations and calculations and for making predictions” (p. 68).

⁹ See the [Math Minds website](#) for a summary of the results and for publications and a free online course for teachers.

group). JUMP students also made significantly more progress on other measure of mathematical achievement.

Based on the interviews with 100 teachers, conducted as part of collaboration with the Vancouver School Board, JUMP Math:

- ✓ Promotes independent thinking,
- ✓ Creates excitement and curiosity,
- ✓ Helps students organize their thinking and articulate what they have learned,
- ✓ Fosters feelings of confidence and a willingness to take risks, and
- ✓ Fosters “a sense of connection and a feeling of belonging to the larger group.

The report found an “overwhelming agreement among teachers that JUMP Math develops among teachers a confidence, self-efficacy and sense that they can do math and even love it.” Another educator wrote: “In my twenty years in education, I have never seen students more excited about math.” (Katie Henderson, Curriculum Supervisor for Morehouse Parish).

Why are these results possible?

Findings of Alfieri et al. (2011) suggest that “unassisted discovery does not benefit learners, whereas feedback, worked examples, scaffolding, and elicited explanations do” (p. 1). In his meta-analysis of educational research, Hattie (2003) found that:

Achievement is more likely to be increased when students invoke learning rather than performance strategies, accept rather than discount feedback, benchmark to difficult rather than easy goals, compare themselves to subject criteria rather than to other students, possess high rather than low efficacy to learning, and affect self-regulation and personal control rather than learned helplessness in the academic situation. (p. 47)

Evidence from cognitive science suggests that mathematics is a subject that should be accessible to every brain. As well, logicians proved that all mathematics can be reduced to steps that are conceptually trivial. Psychologists have shown that virtually any ability can be developed through a process called “deliberate practice.” In agreement with Hattie’s claim, students’ brains work most efficiently when they...

- ✓ ...can struggle or make mistakes without penalty and without feeling they are inferior,
- ✓ ...are given work that looks challenging but isn’t too challenging.

Math Minds researchers, from Werklund School of Education at the University of Calgary, reviewed Canadian and international programs and selected JUMP Math as a partner because of the level of detail and rigour in the JUMP scaffolding. This scaffolding, which makes small conceptual variations visible for teachers, helps learners make discoveries and “critical discernments.” JUMP Math approach is to keep students in a zone of productive struggle, with understanding that the width of this zone is different for novice and expert learners.

Conclusions

Even before the 2020-21 pandemic, educators were worried around the statistics that show how students who fall behind, may not be able to catch up with the more successful peers (Dougherty & Fleming, 2012). Also, that catching up is more likely in the middle school, than at later grade levels. During the pandemic, the gaps between the students increased. Kaffenberger's (2021) modeling suggested that students could lose more than a full year's learning from a three-month closure due to COVID. However, the models also point to possibilities of abrupt mitigation through implementation of aggressive remediation programs.

The JUMP Math vision is that mathematics is interesting in itself, and that the science of learning can guide us reaching out to and engaging all students in learning. These vision statements and the resources, including professional learning opportunities, make us hopeful that efficient support for learners and teachers of mathematics is available.

Questions to discuss with teachers of mathematics

1. Question for Math Leads:

"Your teacher is planning to teach a problem-solving lesson next week using a question like this one. How do you prepare your teacher to plan for teaching this?"

Note: In breakouts groups, listen and note what is shared. Come back to share on slide things like, think about misconceptions, what prior knowledge is needed, what will you do if a student has everything correct, how can you scaffold their learning if they need support or need more of a challenge?

2. After going through the list of resources, ask:

"How might this support your efforts to decrease the gap?"

Note: Opportunities to try these in your schools: If you want to pilot a series of exercises for growth mindset and develop executive functioning, please reach out to us

3. Survey your attendees:

Are you interested in engaging more with us at JUMP math around the following?

- *Trying out Confidence Builders/SEL*
- *Learning more about coaching teachers in JUMP Math*
- *Being a part of a leader advisory group*
- *JUMP in a secondary setting*
- *I want to learn more about _____ financial literacy, how to use essential lessons, math minds, other.*

The list of resources and links

JUMP Math has developed a variety of materials to help you teach math. In this section, we provide links to components and features of the [JUMP Math classroom materials](#):

- ✓ Financial Literacy Lessons
- ✓ Free Essential Lessons for K to 8
- ✓ Professional Learning and Coaching
- ✓ Core Classroom Resources for New Ontario K to 8 Curriculum
- ✓ [Math Minds Course](#).

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